The Annual NASA Space Radiation Summer School
2011 Slide Competition
For The Health Risks of Extraterrestrial Environments (THREE)

Honorable Mention

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Submission on

The Use of Poisson Distribution for Cell Hits

from

Lecture on Radiation Interactions with Matter

Tom Borak (2011)

NASA Space Radiation Summer School

The Use of Poisson Distribution for Cell Hits

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Track structure

$$p[n_H] = \frac{(\overline{n_H})^{n_H} \bullet e^{-\overline{n_H}}}{n_H!}$$

Derivation of Poisson Distribution

In this slide, Dr. Borak was showing the equation of how to calculate the probability of hitting n_H cells. This probability is given by a Poisson Distribution, which is shown in his slide. I would now like to discuss the derivation of Poisson's Distribution.

First, let's start out with a few facts that we know:

1 - $\lim\limits_{n\to\infty}(1+\frac{c}{n})^n = \exp^c$

2 - From combinatorics: $C(n,k) = \frac{n!}{k!(n-k)!}$.

3 - $\lim\limits_{n\to\infty}\frac{C(n,k)}{n^k} = \lim n \to \infty \frac{n!}{k!(n-k)!n^k}$. By reducing the fraction and using limit laws, it can be found that this limit is equal to $\frac{1}{k!}$.

Derivation of Poisson Distribution

4 - Binomial Distribution is used to approximate the distribution of a random variable X with k successes (hits) out of n trials (cells), where $P(X = k) = C(n, k)p^k q^{n-k}$. In this definition, p is the probability of success and q is the probability of failure. However, this computation can be very tedious and it is also known that there is a small probability of multiple hits that is not taken into account with this formula.

Derivation of Poisson Distribution

Suppose that the average number of particles arriving in a interval of length/second is γ so that the distribution of a random variable X with the expectation is $E(X) = \gamma$.

If small intervals of time are taken, we can then consider that the probability of success in a binomial distribution with n trials is $P(success) = \frac{\gamma}{n}$, where n is the number of trials. Therefore,

$$\begin{aligned} P(X = k) &= \lim_{n\to\infty} C(n,k)\left(\frac{\gamma}{n}\right)^k\left(1 - \frac{\gamma}{n}\right)^{n-k} \\ &= \gamma^k \lim_{n\to\infty} C(n,k)\left(\frac{1}{n}\right)^k\left(1 - \frac{\gamma}{n}\right)^{n-k}. \end{aligned} \quad (1)$$

Derivation of Poisson Distribution

Using the limits that we stated in numbers 1-4,

$$
\begin{aligned}
P(X=k) &= \gamma^k\left(\frac{1}{k!}\right)\lim_{n\to\infty}\left(1+\frac{-\gamma}{n}\right)^n \\
&= \gamma^k\left(\frac{1}{k!}\right)\exp^{-\gamma} \\
&= \frac{(\text{ave. number of hits})^{\text{hits}}\cdot\exp^{-\text{ave. number of hits}}}{\text{hits!}} \qquad (2)
\end{aligned}
$$

Fact: The Poisson Distribution was developed by a French mathematician of the nineteenth century. Some applications include radioactive counts, the number of cells that divide in a particular unit of time, number of defects in a length of wire, etc.